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# Coupled channel effects in $\pi\pi$ S-wave interaction

F.Q.Wu and B.S.Zou

*CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China ;*
*Institute of High Energy Physics, CAS, Beijing 100049, China*

We study coupled channel effects upon isospin $I = 2$ and $I = 0$ $\pi\pi$ S-wave interaction. With introduction of the $\pi\pi \to \rho\rho \to \pi\pi$ coupled channel box diagram contribution into $\pi\pi$ amplitude in addition to $\rho$ and $f_2(1270)$ exchange, we reproduce the $\pi\pi$ I=2 S-wave and D-wave scattering phase shifts and inelasticities up to 2 GeV quite well in a K-matrix formalism. For $I = 0$ case, the same $\pi\pi \to \rho\rho \to \pi\pi$ box diagram is found to give the largest contribution for the inelasticity among all possible coupled channels including $\pi\pi \to \omega\omega \to \pi\pi$, $\pi\pi \to K\overline{K} \to \pi\pi$. We also show why the broad $\sigma$ appears narrower in production processes than in $\pi\pi$ scattering process.

*Keywords*: $\pi\pi$ scattering, coupled channel, K-matrix

## 1. INTRODUCTION

As well known isospin $I = 0$ $\pi\pi$ S-wave interaction gives a good place to study the $I = 0$ $J^{pc} = 0^{++}$ particles such as $\sigma$ and glueball. However, to really understand the isoscalar $\pi\pi$ S-wave interaction, one must first understand the isospin I=2 $\pi\pi$ S-wave interaction due to the following two reasons: (1) There are no known s-channel resonances and less coupled channels in I=2 $\pi\pi$ system, so it is much simpler than the I=0 $\pi\pi$ S-wave interaction; (2) To extract I=0 $\pi\pi$ S-wave phase shifts from experimental data on $\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+\pi^- \to \pi^0\pi^0$ obtained by $\pi N \to \pi\pi N$ reactions, one needs an input of the I=2 $\pi\pi$ S-wave interaction.

Up to now, experimental information on the I=2 $\pi\pi$ scattering mainly came from $\pi^+ p \to \pi^+\pi^+ n$ [1] and $\pi^- d \to \pi^-\pi^- pp$ [2] reactions. In previous analyses, the feature of inelasticity $\eta_0^2$ starting to deviate from 1 for energies above 1.1 GeV was often overlooked. Recently, with a K-matrix formalism [3], we show [4] that the feature can be well reproduced by $\pi\pi$-$\rho\rho$ coupled-channel effect. Here we extend the study of the coupled channel effects to the I=0 case which allows much more coupled channels. We also show why the broad $\sigma$ appears narrower in production processes than in $\pi\pi$ scattering process.

## 2. Coupled channel effects in $\pi\pi$ scattering

We follow the $K$-matrix formalism as in Ref.[3]. For $\pi\pi$ scattering, the scattering amplitude can be given in K-matrix formalism as

$$T_{el} = \frac{K}{1 - i\rho K} = K + K\ i\rho\ K + K\ i\rho\ K\ i\rho\ K + \cdots \tag{1}$$

which can be expressed diagrammatically as in Fig.1 for $\pi\pi$ scattering at low energies with only one opening channel.

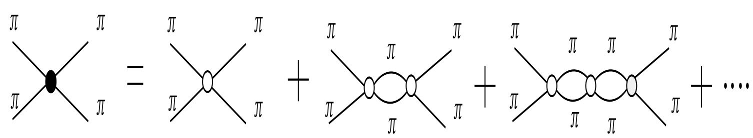

Fig. 1. Diagrammatic expression for $\pi\pi$ scattering in K-matrix formalism

For the two-channel case, the two-dimensional $K$ matrix and phase space $\rho(s)$ matrix are

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}, \qquad \rho(s) = \begin{pmatrix} \rho_1(s) & 0 \\ 0 & \rho_2(s) \end{pmatrix}, \tag{2}$$

with i=1,2 representing $\pi\pi$ and $\rho\rho$ channel, respectively. Ignoring the interaction between $\rho\rho$, we have $K_{22} = 0$; then

$$T_{11} = \frac{K_{11} + iK_{12}\rho_2 K_{21}}{1 - i\rho_1(K_{11} + iK_{12}\rho_2 K_{21})}, \tag{3}$$

where $iK_{12}\rho_2 K_{21}$ corresponds to the $\rho\rho$ on-shell contribution[5] of the $\pi\pi \to \rho\rho \to \pi\pi$ box diagram as shown in Fig.2.

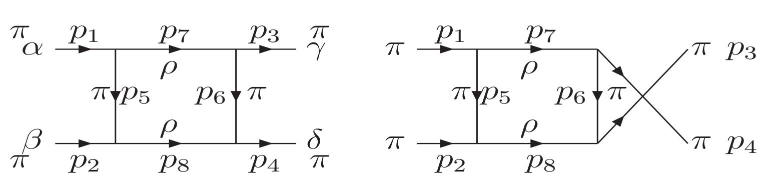

Fig. 2. The $\pi\pi \to \rho\rho \to \pi\pi$ box diagrams

With $K_{11}$ including contribution from $t$-channel $\rho$ and $f_2(1270)$ exchange, we found that the basic features of I=2 $\pi\pi$ S-wave phase shifts and inelasticities are well reproduced as shown in Fig.3. For details, see Ref.[4].

With the success in reproducing the $I = 2$ $\pi\pi$ S-wave scattering, we extend our study of the coupled channel effects to the isospin I=0 $\pi\pi$ scattering. Here

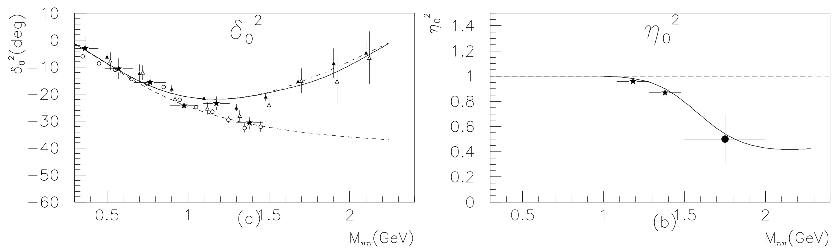


Fig. 3. The $I = 2$ $\pi\pi$ $S$-wave ($\delta_0^2$, $\eta_0^2$) phase shifts and inelasticities. Data are from Ref.[1,2,3]. The solid curves represent the total contribution of $\rho$, $f_2$ exchange and the box diagram; dot-dashed curves from $\rho$ and $f_2$ exchange; dashed curves from t-channel $\rho$ exchange only.

in additional to the box diagrams shown in Fig. 2, there are more other coupled channels such as $\pi\pi \to \omega\omega \to \pi\pi$, $\pi\pi \to \sigma\sigma \to \pi\pi$ and $\pi\pi \to K\overline{K} \to \pi\pi$ as shown in Fig.4.

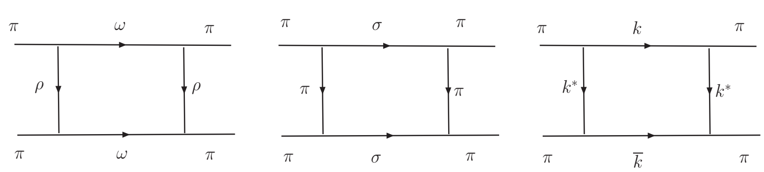

Fig. 4. The $\pi\pi \to \omega\omega \to \pi\pi$, $\pi\pi \to \sigma\sigma \to \pi\pi$ and $\pi\pi \to K\overline{K} \to \pi\pi$ box diagrams for I=0 case.

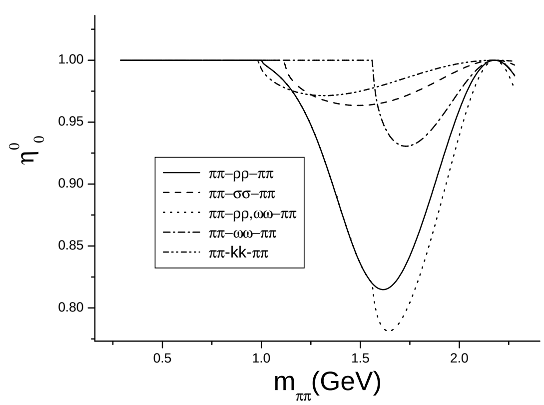


Fig. 5. The $I = 0$ $\pi\pi$ S-wave inelasticity $\eta_0^0$ without including s-channel resonances in the $I = 0$ amplitude. "$\pi\pi - \rho\rho - \pi\pi$" means including $\pi\pi - \rho\rho - \pi\pi$ box diagram; "$\pi\pi - \rho\rho, \omega\omega - \pi\pi$" means using three-dimensions K matrix to couple $\pi\pi, \rho\rho$ and $\omega\omega$ channels together.

To demonstrate the significance of coupled channel effects in the I=0 $\pi\pi$ scattering, we do not include any s-channel resonances in the $I = 0$ amplitude, just introduce $\pi\pi \to \rho\rho \to \pi\pi$ ,$\pi\pi \to \omega\omega \to \pi\pi$, $\pi\pi \to K\overline{K} \to \pi\pi$ and $\pi\pi \to \sigma\sigma \to \pi\pi$ box diagrams respectively into the $I = 0$ amplitude which includes t-channel $\rho$ and $f_2(1270)$ exchange [3]. The result shows $\pi\pi \to \rho\rho \to \pi\pi$ (solid line in Fig. 5) gives the most important contribution. We also use three-dimensions K matrix to couple $\pi\pi, \rho\rho$, and $\omega\omega$ channels together. The result is shown by the dotted line in Fig.5.

## 3. $\pi\pi$ S-wave interaction in production processes

$\pi\pi$ production processes can be regarded as a special case for coupled channels. For the $\pi\pi$ S-wave interaction in production processes, it can be expressed diagrammatically as in Fig.6.

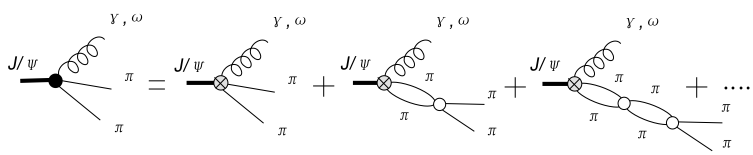


Fig. 6. Diagrammatic expression for $\pi\pi$ scattering in K-matrix formalism

Compared with elastic scattering shown in Fig.1, the only difference is the first interaction vertex. So the production amplitude can be expressed similar to Eq.(1) as

$$T_{prod} = \frac{P}{1 - i\rho K} = P + P\ i\rho\ K + P\ i\rho\ K\ i\rho\ K + \cdots \tag{4}$$

For the I=0 $\pi\pi$ S-wave scattering at low energies ($\leq 2m_K$, the K-matrix and its corresponding elastic scattering amplitude $T_{el}$ can be well determined by the $\pi\pi$ scattering phase shift data [6,7,8]. The solid line in Fig.7 shows a solution [9] for $|\rho_1 T_{el}|^2$ from fitting the well-known CERN-Munich $\pi\pi$ scattering data [6,7]. The solution has a T-matrix pole at $571 - i420$ MeV for the broad $\sigma$ which provides the broad background for two narrow dips caused by its interference with the narrow $f_0(980)$ and $f_0(1500)$. With the same K-matrix, if we assume P-matrix to be 1 for $\pi\pi$ production, we can get $|\rho_1 T_{prod}|^2$ as shown by the dashed line in Fig.7. A much narrower peak at lower energy is appearing although in fact it has the same broad pole as in the $\pi\pi$ elastic scattering process. This gives a clear demonstration why the broad $\sigma$ appears narrower in production processes than in $\pi\pi$ scattering process. The reason is $T_{prod} \sim T_{el}/K$ here. This has also been noted by Ref.[10] in a slightly different language.

Some recent analyses [11,12] of various $\pi\pi$ production processes gave a much narrower $\sigma$ pole than Ref.[9,13] from $\pi\pi$ scattering. A reason is that it is assumed a direct production of $\sigma$ with production vertex $P = 1/(m_\sigma^2 - m_{\pi\pi}^2)$ instead of

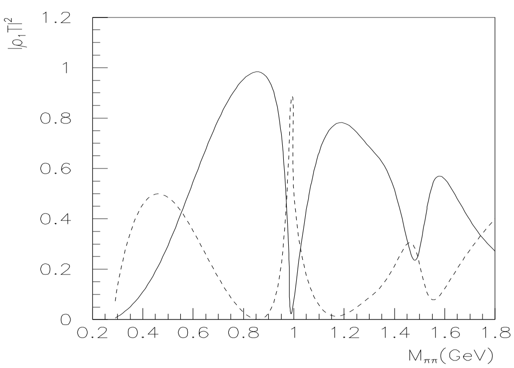


Fig. 7. Amplitude squared for $\pi\pi$ S-wave in production process (dashed curve) compared with in elastic scattering (solid curve), assuming P=1 and K from Ref.[9].

considering the $\sigma$ due to final state $\pi\pi$ scattering with $P = 1$ or some smooth function of $m_{\pi\pi}$. With the same production data but with different production vertex $P$, one will get different $\sigma$ pole. See Refs.[11,14] for an example.

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